Nontrivial critical crossover between directed percolation models: Effect of infinitely many absorbing states

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At nonequilibrium phase transitions into absorbing (trapped) states, it is well known that the directed percolation (DP) critical scaling is shared by two classes of models with a single (S) absorbing state and with infinitely many (IM) absorbing states. We study the crossover behavior in one dimension, arising from a considerable reduction of the number of absorbing states (typically from the IM-type to the S-type DP models) by following two different (excitatory or inhibitory) routes which make the auxiliary field density abruptly jump at the crossover. Along the excitatory route, the system becomes overly activated even for an infinitesimal perturbation and its crossover becomes discontinuous. Along the inhibitory route, we find a continuous crossover with universal crossover exponent $\phi \simeq 1.78(6)$, which is argued to be equal to $\nu_\parallel$, the relaxation time exponent of the DP universality class on a general footing. This conjecture is also confirmed in the case of the directed Ising (parity-conserving) class. Finally, we discuss the effect of diffusion on the IM-type models and suggest an argument why diffusive models with some hybrid-type reactions should belong to the DP class.

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I. INTRODUCTION

The directed percolation (DP) has been studied extensively as one of the typical dynamic critical phenomena far from equilibrium [1]. Nonequilibrium phase transitions of systems with a unique absorbing state are found to belong to the DP class if no symmetry or conservation of the order parameter is present [2]. Even systems with infinitely many (IM) absorbing states such as the pair contact process (PCP) [3] are believed to share the critical behavior with the DP [3–5], but the full theoretical understanding of the IM-type DP ($\mathrm{DP_{IM}}$) models is still lacking. For example, the perturbative renormalization group study based on the phenomenological action introduced in Ref. [6] predicts that the PCP should belong to the dynamic percolation rather than the DP class [7] even though the phenomenological action itself is believed to describe the PCP correctly [6]. Another issue is on the spreading dynamics which shows the nonuniversal behavior [8–10].

One may reduce the number of absorbing states significantly by introducing particle diffusion [PCP with diffusion (PCPD)] [11]. Surprisingly, the PCPD has caused serious turmoil, which could not be calmed down in spite of extensive numerical [12–14] and analytical [15] studies. The answer seems to be one of two possibilities: The PCPD belongs to the DP class with a long transient or forms a new universality class distinct from known universality classes to date. As an attempt to resolve the issue, the present authors suggested two different approaches to the PCPD.

First, introducing the dynamic perturbation which is implemented by biased hopping, we showed that the one-dimensional PCPD with biased diffusion [driven PCPD (DPCPD)] exhibits a critical scaling distinct from the unbiased PCPD, but instead shares the critical behavior with the two-dimensional PCPD without bias (dimensional reduction) [16,17]. Since all known DP models are robust against biased diffusion [16], a DPCPD-type variant can serve as a litmus test for PCPD scaling [18]. Second, we studied the crossover behavior from the PCPD to the DP by introducing single-particle annihilation and branching reactions and showed that there are diverging crossover scales with the universal nontrivial crossover exponent [19]. These results provided another piece of evidence that the PCPD is distinct from the DP.

In this paper, we study the crossover behavior from the $\mathrm{DP_{IM}}$ to the DP in order to understand better the difference between these two "equivalent" (DP) universality classes. Actually, it would be absurd to talk about the crossover between two models belonging to the identical universality class (see Sec. II). However, the $\mathrm{DP_{IM}}$ models differ from the DP models in regard to the "nonorder" parameter at the transitions, which shows a discontinuous singularity at the crossover. This singularity induces a well-defined and nontrivial crossover from the $\mathrm{DP_{IM}}$ to the DP, more generally a crossover arising from a considerable reduction of the number of absorbing states (even between two different $\mathrm{DP_{IM}}$).

We find two distinct crossover behaviors depending on the routes to reduce the number of absorbing states. Infinitesimal inclusion of an "excitatory" process (such as single-particle branching) makes the system overly active, which gives rise to a discontinuous crossover (see Sec. III), while the opposite "inhibitory" route (such as single-particle annihilation) reveals a continuous crossover with the nontrivial crossover exponent $\phi \simeq 1.78(6)$. We find that this crossover exponent is universal for various kinds of models including the PCP and the triplet contact process (TCP). We argue that the crossover exponent is not independent but equal to $\nu_\parallel$, the relaxation time exponent of the DP universality class on a general footing. This conjecture is also confirmed in the case of the directed Ising (parity-conserving) class (see Sec. IV).

TABLE I. d-dimensional reaction-diffusion processes of single species with hard-core exclusion and their rates.

Diffusion	$A\emptyset \leftrightarrow \emptyset A$	With rate D/d
Pair annihilation	$AA \to \emptyset\emptyset$	With rate λ/d
Coalescence	$AA \to A\emptyset$	With rate $\eta/(2d)$
Coalescence	$AA \to \emptyset A$	With rate $\eta/(2d)$
Death	$A \to \emptyset$	With rate ξ
Branching	$\emptyset A \to AA$	With rate $\sigma/(2d)$
Branching	$A\emptyset \to AA$	With rate $\sigma/(2d)$

The PCPD *per se* can be considered as one of the crossover models from the $\mathrm{DP_{IM}}$ by allowing diffusion, which reduces the number of absorbing states considerably (from exponentially many to linearly many absorbing states with respect to system size). However, the crossover study from the PCP to the PCPD does not give any useful information on the $\mathrm{DP_{IM}}$, because the particle diffusion makes the system more active (excitatory) and the crossover turns out to be discontinuous.

Finally, we study the crossover from the diffusive reaction models to the DP. Such an example is the crossover from the PCPD to the DP studied in Ref. [19]. In Sec. V, the hybrid-type models with diffusion (tp12: $2A \to \emptyset, 3A \to 4A$) [12] are perturbed by adding a single-particle annihilation process ($A \to \emptyset$) and its crossover to the DP is investigated. These hybrid-type models (where the branching process is of higher order than the annihilation process) are numerically known to belong to the DP class. As expected, there is no nontrivial crossover and the DP critical line emanates "linearly" from the hybrid-type model point. We suggest an argument why these models should belong to the DP rather than a PCPD-type nontrivial class.

II. CROSSOVER BETWEEN THE IDENTICAL UNIVERSALITY CLASS

This section considers a d-dimensional stochastic system of hard-core particles with the dynamics summarized in Table I. In Ref. [20], it is shown that two different stochastic systems specified by tilded and untilded rates are equivalent if the transition rates satisfy the relations

$$\tilde{\sigma} = \mu\sigma, \quad 2\tilde{D} + \tilde{\xi} = 2D + \xi,$$

$$\tilde{D} + \frac{\tilde{\sigma}}{2} = D + \frac{\sigma}{2},$$

$$\tilde{\lambda} + \frac{\tilde{\eta}}{2} + \frac{\tilde{\sigma}}{2} = \frac{1}{\mu}\left(\lambda + \frac{\eta}{2} + \frac{\sigma}{2}\right),$$

$$\tilde{\lambda} + \tilde{\eta} + \tilde{\sigma} - 2\tilde{D} = \lambda + \eta + \sigma - 2D, \tag{1}$$

where μ is a transformation constant and all transition rates should be non-negative. Any correlation function for the tilded system is simply related to that for the untilded system. For instance, the particle density ρ at time t becomes [20]

$$\rho(D,\lambda,\eta,\xi,\sigma;\rho_0,t) = \frac{1}{\mu}\tilde{\rho}(\tilde{D},\tilde{\lambda},\tilde{\eta},\tilde{\xi},\tilde{\sigma};\tilde{\rho}_0,t) \tag{2}$$

if the initial density of two systems has the relation $\tilde{\rho}_0 = \mu\rho_0$. Needless to say, both ρ_0 and $\tilde{\rho}_0$ should lie between 0 and 1. Although equivalence is shown only for the one-dimensional system in Ref. [20], Eq. (1) is generally true for any d-dimensional systems, which can be easily shown by the same technique developed in Ref. [20].

Now consider the branching annihilating random walks with one offspring (BAW1) [21] which corresponds to the model with $\xi = 0$ in Table I. The parameters used in Ref. [21] in one dimension are $D = p/2$, $\lambda = p$, and $\eta = \sigma = 1 - p$ with the tuning parameter p. If $w \equiv \mu - 1$ is very small and non-negative, the solution of Eq. (1) up to the order of w is

$$\tilde{D} - D = -\frac{w}{2}\sigma, \quad \tilde{\sigma} - \sigma = w\sigma, \quad \tilde{\xi} = w\sigma,$$

$$\tilde{\lambda} - \lambda \simeq -w(\eta + 2\lambda), \quad \tilde{\eta} - \eta \simeq w(\eta + 2\lambda - 2\sigma). \tag{3}$$

If w is sufficiently small, it is always possible to associate the BAW1 with a stochastic process with spontaneous death in an equivalent way with all non-negative rates. Since the BAW1 in high dimensions is also known to have a nontrivial transition point [22], the following discussion is valid in any spatial dimensions. The transition points for stochastic systems with small w can be always calculated exactly from Eq. (1) [approximately from Eq. (3)] if the transition point of the BAW1 is given.

The conclusions from the above analysis are twofold. First, it is clear from Eq. (3) that the phase boundary (critical line) should meet the BAW1 transition point *linearly* with finite slope as w vanishes. This implies that there is no additional singularity involved near $w = 0$, which is fully expected from the crossover between models with identical universality classes. If one defines the crossover exponent ϕ from the shape of the critical line near $w = 0$ (see Sec. III), one can get $\phi = 1$. Second, the critical decay of the density is given by $\tilde{\rho}_c(t) = (1 + w)\rho_c(t)$ from Eq. (2), which implies that there is no diverging crossover time scale for small w.

Since the introduction of spontaneous death does not change the structure of the absorbing phase space (single absorbing state), let alone the universality class, the above analysis is in good harmony with the naive expectation as to the "crossover" between two models belonging to identical classes. In the next section, however, we will show that a substantial change of the absorbing phase space without affecting the universality class will trigger a nontrivial crossover.

III. CROSSOVER FROM THE $\mathrm{DP_{IM}}$ TO THE DP

A. Crossover model from the PCP to the DP

Unlike the BAW1, the PCP is the prototype of $\mathrm{DP_{IM}}$ models with exponentially many absorbing states. By introducing single-particle reactions to the PCP, the number of absorbing

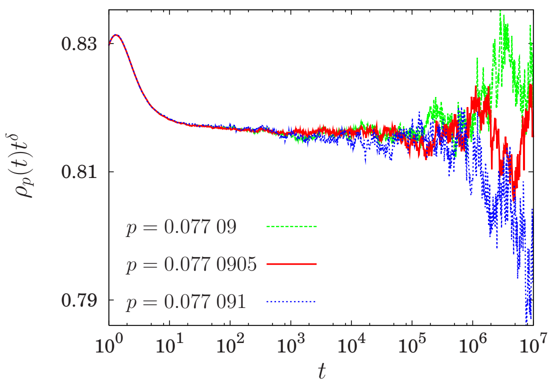

FIG. 1. (Color online) Semilogarithmic plot of $\rho_p(t)t^\delta$ vs t of the PCP near criticality with δ=0.1595 the exponent of the DP. Since the upper (lower) curve veers up (down), we estimate the critical point as p_0=0.077 090 5(5) with the error in the last digit by 5.

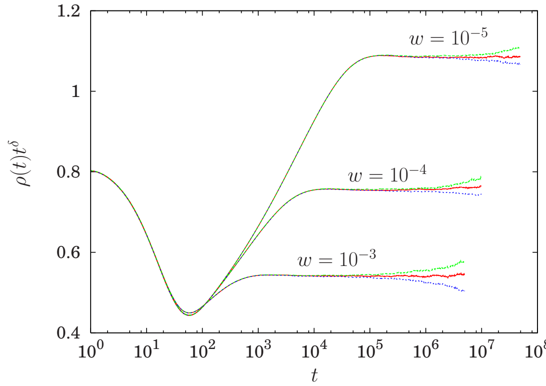

FIG. 2. (Color online) Plots of $\rho(t)t^\delta$ vs t for w=10^{-3}, 10^{-4}, and w=10^{-5} at q=1 close to the critical points in semilogarithmic scales. Again, δ assumes the DP value. The curves in the middle correspond to p=0.1451, 0.1448, and 0.1448 (from bottom to top), respectively, and the value of p's of other two curves are ±0.0001 off from the middle value. As w becomes smaller, the relaxation time becomes larger though the critical point does not change much and approaches $p\simeq$0.1448.

states changes drastically from infinity to 1. This section shows that this qualitative change is reflected in the singular behavior of the phase boundary close to the PCP transition point in one dimension.

The dynamics of the model is summarized as

$$AA \xrightarrow{p} \emptyset\,\emptyset, \quad \left.\begin{matrix} AA\emptyset \\ \emptyset AA \end{matrix}\right\} \xrightarrow{(1-p)/2} AAA, \tag{4a}$$

$$\left.\begin{matrix} A\,\emptyset \\ \emptyset A \end{matrix}\right\} \xrightarrow{(1-q)w/2} \emptyset\,\emptyset, \quad \left.\begin{matrix} A\,\emptyset \\ \emptyset\,\,A \end{matrix}\right\} \xrightarrow{qw/2} AA, \tag{4b}$$

where $0 \leq q \leq 1$. For the PCP at w=0, any configuration without a pair of neighboring particles (a mixture of isolated particles and vacant sites) is absorbing and its number grows exponentially with system size. The order parameter of the PCP is the pair density (the number density of AA pairs), and the particle density field is *auxiliary* which is finite even in the absorbing phase. At nonzero w, an isolated particle becomes active and only the vacuum becomes the true absorbing state. In this case, the particle density is usually adopted as the order parameter and the pair density scales in the same way as the particle density.

Figure 1 locates the transition point of the PCP (w=0) at p_0=0.077 090 5(5) by exploiting the critical decay of the pair density as $\rho_p(t) \sim t^{-\delta}$ with δ the critical exponent of the DP class whose accurate value can be found in [23]. In numerical simulations, the system size is L=2^{18} and the number of independent samples is 750, 1500, and 400 for the data in the active, critical, and absorbing phases, respectively. The flatness of $\rho_p(t)t^\delta$ over four logarithmic decades in time confirms the solid DP critical scaling of the PCP.

At finite w, the model still belongs to the DP class irrespective of q. Unlike the PCPD-to-DP crossover model in Ref. [19], however, the critical lines show two completely different singular behaviors, depending on the value of q. For large q, the activity of the system is enhanced by additional single-particle reaction processes (excitatory process) and the system becomes overly activated even with infinitesimal w. The critical line does not converge to the PCP critical point as w decreases to zero (w=0^+) and shows a discontinuous jump. On the other hand, for small q, the system activity is suppressed (inhibitory process) and the system becomes more inactive. The critical line nicely converges to the PCP critical point and shows a continuous crossover with a nontrivial crossover exponent.

B. Excitatory route q=1

First, we choose the q=1 case as a typical excitatory route of the crossover from the DP_{IM} to the DP. As shown in Fig. 2, the critical line approaches $p\simeq$0.1448 as w approaches zero, which is by far above the critical value of the PCP ($p_0\simeq$0.077). So there is a big jump of the critical line at w=0. The discontinuity can be understood as follows: Consider a system with p slightly above the PCP critical point p_0 and $0 < w \ll \tau^{-1}$ where τ is the relaxation time which is finite off criticality. Then the single-particle branching event ($A \to 2A$) with characteristic time w^{-1} occurs effectively after the system falls into one of the PCP absorbing states in which the isolated particle density is finite. Since the branching event creates a new pair, the system is reactivated and performs damage-spreading-type "defect dynamics" for some time proportional to τ and again falls into one of the PCP absorbing states. This defect dynamics continues forever with the period of time w^{-1}. As the particle density is finite (and quite large) even in the PCP absorbing states, the time-averaged particle density in this iterated process should be finite in this region of the phase diagram. This implies that the continuous absorbing phase transition at infinitesimal w into vacuum should occur way above p_0, which is consistent with our findings. Note that the discontinuity in the auxiliary field (particle) density is crucial in this crossover.

Actually, the same argument can be applied to the crossover from the PCP to the PCPD. We can introduce the dif-

TABLE II. Critical points of the model with dynamics of Eq. (4) for some values of w's at $q=0$. The numbers in the parentheses indicate the error of the last digits.

w	$p_c(w)$
0	0.077 0905(5)
10^{-5}	0.077 002(3)
5×10^{-5}	0.076 885(3)
10^{-4}	0.076 784(4)
2×10^{-4}	0.076 642(2)
3×10^{-4}	0.076 530(5)
4×10^{-4}	0.076 432(2)
5×10^{-4}	0.076 345(2)
6×10^{-4}	0.076 264(1)
10^{-3}	0.075 988(4)

fusion rather than the single-particle reactions and again consider p slightly above p_0. Let ρ_0 denote the isolated particle density at the PCP absorbing states; then, the characteristic length scale between isolated particles is $1/\rho_0$. If $0 < D\rho_0^2 \ll \tau^{-1}$ with the diffusion constant D, the "defect dynamics" will continue again indefinitely for small D. So the phase boundary in the D-p plane should have a discontinuity at $D=0$.

C. Inhibitory route $q=0$

Let us turn to the crossover model with $q=0$, which should represent a typical inhibitory route. Table II summarizes the critical points of the model for some w's at $q=0$, and the corresponding phase boundary is plotted in Fig. 3. Unlike the previous case, the reactive phase shrinks continuously with the rate of additional single-particle annihilation process and the phase boundary is continuous. The usual

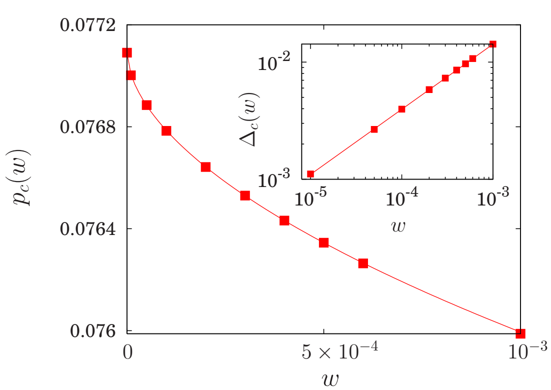

FIG. 3. (Color online) Phase boundary of the model of Eq. (4) at $q=0$ in the w-p plane. Symbols locate the numerically estimated critical points. The error of the critical point is smaller than the symbol size. The curve shows the least-squares-fit result of the phase boundary. The absorbing (active) phase is above (below) the curve. Inset: the same but the vertical axis is $\Delta_c(w)$ in log-log scale. The slope corresponds to the inverse of the crossover exponent which is estimated as $\phi=1.78(6)$.

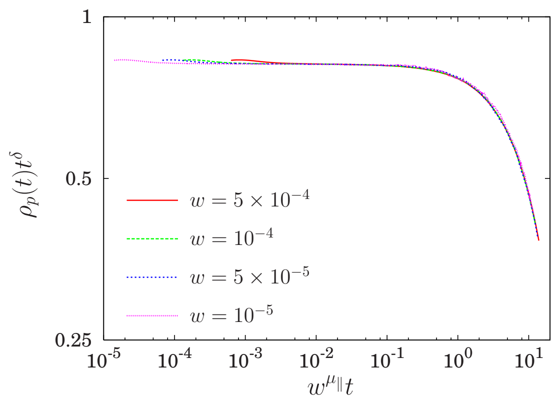

FIG. 4. (Color online) Log-log plot of the scaling function, Eq. (6), for the PCP crossover model using $\delta=0.1595$ and $\mu=0.97$. All curves are collapsed into a single curve.

analysis method can be applied to this case [24]. If we define $\Delta=(p_0-p)/p_0$ and $\Delta_c(w)=[p_0-p_c(w)]/p_0$, the phase boundary is well fitted by $\Delta_c \sim w^{1/\phi}$ with $\phi^{-1}=0.56(2)$ or $\phi=1.78(6)$; see the inset of Fig. 3. Let us assume the existence of a well-defined crossover scaling which is described by the scaling function [24]

$$\rho_p(\Delta,w;t)=t^{-\delta}\mathcal{F}(\Delta^{\nu_\parallel}t,w^{\mu_\parallel}t), \tag{5}$$

where ρ_p is the pair density and $\mu_\parallel=\nu_\parallel/\phi$ with ϕ estimated in the above. We examine whether the scaling function in Eq. (5) correctly describe the crossover near the PCP critical point.

First, we measure the pair density for various w's at $\Delta=0$. From the scaling ansatz (5), the pair density at $\Delta=0$ should collapse as

$$t^{\delta}\rho_p(t)=\mathcal{G}(w^{\mu_\parallel}t). \tag{6}$$

With $\mu_\parallel\simeq0.97$, all curves for the pair density are collapsed into a single curve as Fig. 4 shows. Next, we take $\Delta=\Delta_c(w)$ along the critical line. Since $\Delta_c(w)\simeq w^{1/\phi}$ and $\nu_\parallel/\phi=\mu_\parallel$, the scaling function should take the form

$$t^{\delta}\rho_p[\Delta_c(w);t]=\mathcal{H}(w^{\mu_\parallel}t), \tag{7}$$

where $\mathcal{H}(x)$ approaches a constant as $x\to\infty$. In Fig. 5, all curves at different critical points collapse well into a single curve. Hence we conclude that the scaling function, Eq. (5), correctly describes the crossover behavior from the $\mathrm{DP}_{\mathrm{IM}}$ to the DP.

Since the models at $w=0$ and at $w\neq0$ belong to the same DP universality class, it is natural to ask what the origin is of such a nontrivial singularity near the PCP critical point. The inset of Fig. 5 gives a hint to this question, which shows that the scaling function $\mathcal{H}$ (the amplitude of the critical decay) does not approach the PCP value as w goes to zero; i.e., it is not continuous at $w=0$. The discontinuity in this amplitude must originate again from the discontinuity in the auxiliary field (particle) density. One can see it directly from the behavior of the particle density (ρ). Unlike the pair density, ρ cannot be described by the scaling function (5). Consider again the case at $\Delta=0$ and nonzero w. For any finite value of

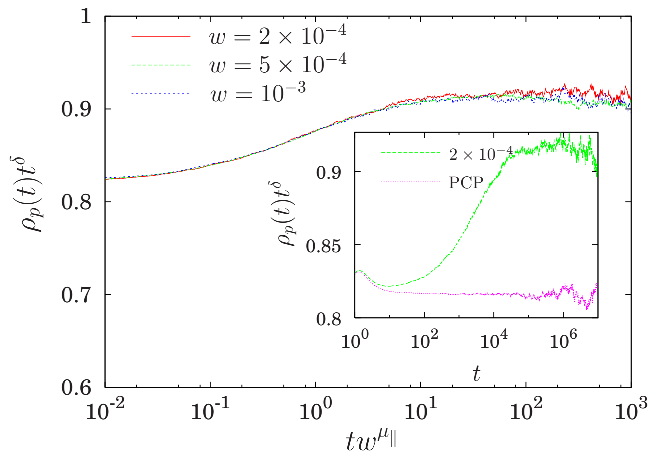

FIG. 5. (Color online) Scaling plot of $\rho_p(t)t^{\delta}$ vs $w^{\mu_\parallel}t$ with $\delta=0.1595$ and $\mu_\parallel=0.97$ at criticality in semilogarithmic scales. Inset: scaling function $\mathcal{H}$ at finite w and $w=0$. The asymptotic value of $w=0$ is different from the $w\to 0$ limit.

w in the thermodynamic limit, $\rho(t)$ approaches to zero as $t\to\infty$; see Fig. 6. On the other hand, the model at $\Delta=w=0$ (the critical PCP) has a nonzero density of $\rho(t)$ as $t\to\infty$. In other words, the $w\to 0$ limiting process is different from the $w=0$ model itself in regard to the auxiliary field density.

To check the universality of the crossover exponent, we study the modified PCP with the replacement of $2A\to 3A$ with $3A\to 4A$ in Eq. (4) which is the model of Eq. (9) with no diffusion ($D=0$). This model also has infinitely many absorbing states and belongs to the $\mathrm{DP_{IM}}$ class. By introducing single-particle reactions ($w\neq 0$), the same crossover behavior is found as the above (data not shown).

D. From the $\mathrm{DP_{IM}}$ to the $\mathrm{DP_{IM}}$

We also study more general crossover behavior from one $\mathrm{DP_{IM}}$ to another $\mathrm{DP_{IM}}$ with a considerably reduced number of absorbing states. To be specific, we consider the TCP and its crossover model by introducing the $2A\to A$ process with-

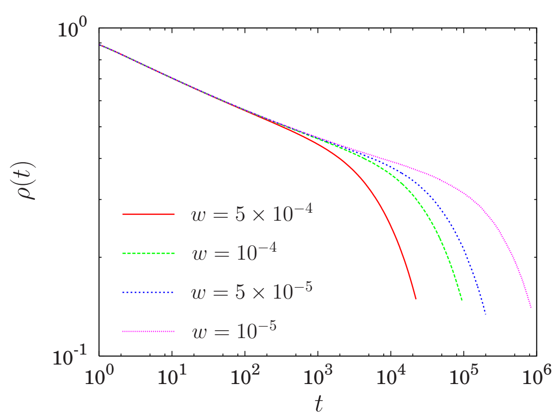

FIG. 6. (Color online) Log-log plot of the particle density for some finite w's at $\Delta=0$. For any finite value of w, the particle density $\rho(t)$ will go to zero, which is clearly different from the $w=0$ case.

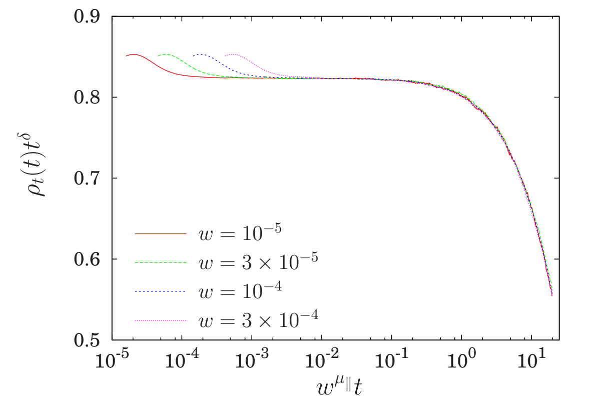

FIG. 7. (Color online) Scaling plot of $\rho_t(t)t^{\delta}$ vs $w^{\mu_\parallel}t$ with the same exponents in Fig. 4 for the crossover TCP model of Eq. (8) in semilogarithmic scales. As in Fig. 4, all curves are collapsed into a single curve.

out spontaneous death. The TCP with pair dynamics is defined as

$$AAA \xrightarrow{p} \emptyset\emptyset\emptyset, \quad \left.\begin{matrix} AAA\emptyset \\ \emptyset AAA \end{matrix}\right\} \xrightarrow{(1-p)/2} AAAA,$$

$$AA\emptyset \xrightarrow{w/2} A\emptyset\emptyset, \quad \emptyset AA \xrightarrow{w/2} \emptyset\emptyset A. \tag{8}$$

The above model has infinitely many absorbing states, but with nonzero w the number of absorbing states is greatly reduced. At $w=0$, there is again a jump in the auxiliary field density (here, the pair density). We found that the critical point for the TCP at $w=0$ is $p_c=0.036\,865(5)$, exploiting the DP critical scaling (data not shown). Figure 7 shows the scaling plot of the triplet density ρ_t in the same way as in Fig. 4. We also measured the crossover exponent from the phase boundary and found the same exponent (data not shown).

Hence we conclude that there is well-defined and universal crossover scaling from the $\mathrm{DP_{IM}}$ to the DP which is mediated by the significant reduction of the number of absorbing states. The discontinuity in the auxiliary field plays a crucial role in this nontrivial crossover.

IV. CONJECTURE ON THE CROSSOVER EXPONENT ϕ

The crossover exponent from the $\mathrm{DP_{IM}}$ to the DP is estimated as $\phi=1.78(6)$. Since this crossover occurs between the same universality class, we are suspicious that ϕ may not be independent but related to well-known DP critical exponents. Actually, we argue that the crossover exponent is given by the DP relaxation time exponent $\phi=\nu_\parallel\simeq 1.733$ (or $\mu_\parallel=1$) which is compatible with the numerical estimation within error. The reason is as follows: Take the model of Eq. (4) at $q=0$. The critical line should be determined by the competition between the single-particle annihilation process ($A\to\emptyset$) parametrized by w and the multiparticle (pair) reaction process ($2A\to\emptyset$ or $3A$) parametrized by $\Delta\sim(p_0-p)$. We expect that both events should appear at the same time scale along the critical line to balance off each other. Since the single-particle (auxiliary field) density is finite at the PCP (DP_{IM}) point, the time scale for $A \to \emptyset$ should be simply proportional to w^{-1}. The time scale for the pair reaction process should be given by the relaxation time scale $\tau \sim \Delta^{-\nu_\parallel}$. Consequently, the critical line is determined as $\Delta_c \sim [p_0 - p_c(w)] \sim w^{1/\nu_\parallel}$, which yields $\phi = \nu_\parallel$ and equivalently $\mu_\parallel = 1$.

Considering the crossover between the DP models with a unique absorbing state, the time scale for the process parametrized by w is proportional to $w^{-\nu_\parallel}$ like the other competing process because the auxiliary field (particle) density is also vanishing critically as w decreases to zero. In this case, we get $\phi = 1$ and equivalently $\mu_\parallel = \nu_\parallel$, which is consistent with our result in Sec. II.

Since our argument for the crossover exponent can be applied to any universality class, we check its validity through studying the similar type crossover between the directed Ising (DI) class models [25]. Consider a one-dimensional system with two species—say, A and B. Between the same species, hard-core exclusion is enforced, but different species can reside at the same site. The dynamic rules are as follows: The dynamics always starts with an A particle. A randomly chosen A particle can hop to one of nearest neighbors with probability p. If two A particles meet at the same site by hopping, both particles are removed with probability $\frac{1}{2}$. If this annihilation attempt fails, the particle goes back to the original site. With probability $1-p$, a B particle is generated at the same site occupied by the chosen A particle. If that site is already occupied by another B particle, the two B particles transmute to two A particles which will be placed at two nearest-neighbor sites. If any of transmuted A particles is placed at the site already occupied by another A particle, both particles are annihilated immediately. In summary, $2A \to \emptyset$, $A \to A + B$, and $2B \to 2A$ processes are allowed with A-particle diffusion.

Since B particles are not allowed to hop, the system is inactive without an A particle but only with B particles. The number of the absorbing states grows exponentially with system size, and the auxiliary field (B-particle) density is finite at the absorbing transition. Besides, the number of A particles is conserved modulo 2, which is the characteristic of the DI (or parity-conserving) class.

As in Sec. III, we study the crossover by introducing spontaneous annihilation of B particles (inhibitory route) with rate w. As expected, we find the DI critical scaling for both $w=0$ and $w \neq 0$ cases. We locate the critical line by exploiting the known DI critical exponents [25], which is summarized in Table III for some w's. Since the critical exponent $\nu_\parallel$ of the DI class ($\simeq 3.25$) is much larger than that of the DP ($\simeq 1.733$), the accuracy of the critical points in Table III is worse than that for the DP cases. From these data, one can estimate the crossover exponent $1/\phi_{DI}$ as 0.34(4) which should be compared with $1/\nu_\parallel$ of the DI class ($\simeq 0.31$). Hence we conclude that our argument for $\phi = \nu_\parallel$ also applies to the crossover from the IM-type DI to the DI models.

V. DIFFUSION EFFECT

This section studies how the crossover scaling is affected if particles are allowed to hop in models considered in Sec. III. With single-particle diffusion, the PCP becomes the PCPD where the particle (auxiliary field) density as well as the pair density vanishes at criticality even without any single-particle reaction process. The crossover from the PCPD to the DP caused by including single-particle reactions has been studied previously by the present authors [19] where the value of the crossover exponent is reported as $1/\phi = 0.58(3)$. This value is quite close to that obtained for the crossover from the DP_{IM} to the DP in Sec. III. This similarity may mislead one to jump to the wrong conclusion that the critical natures of the PCP and PCPD are equivalent. However, one should remember that the origin of the nontrivial crossover from the PCP to the DP lies in the finiteness of the auxiliary field density at the PCP critical point, while it vanishes at the PCPD critical point. So, if the PCPD belongs to the DP class, then one should expect a trivial crossover with $\phi = 1$. On this account, our finding of the nontrivial crossover in [19] supports the conjecture that the PCPD class is distinct from the DP class [26]. Hence it is likely that the similarity of two crossover exponents is a mere coincidence.

TABLE III. Critical points of the DI crossover model for some values of w's (see the text). The numbers in the parentheses indicate the error of the last digits.

w	$p_c(w)$
0	0.4518(1)
10^{-4}	0.4443(2)
3×10^{-4}	0.4405(2)
10^{-3}	0.4353(1)
3×10^{-3}	0.4276(1)

The distinction of the PCPD from the DP can also be evidenced by the study of the crossover model with the hybrid-type reaction dynamics $2A \to 0$ and $3A \to 4A$. Without diffusion, this model belongs to the DP_{IM} and its crossover to the DP induced by inclusion of single-particle reactions was studied in Sec. III. Interestingly, it is numerically shown [12] that the diffusion is irrelevant in this model (called tp12) and its DP nature is intact, in contrast to the PCP case.

We study the crossover behavior of the diffusive tp12 to the DP via inclusion of single-particle reactions to confirm its DP nature and also to compare with the PCPD case. The dynamics of the model is summarized as

$$AA \xrightarrow{p} \emptyset\emptyset, \quad \left.\begin{matrix} AAA\emptyset \\ \emptyset AAA \end{matrix}\right\} \xrightarrow{(1-p)/2} AAAA,$$

$$\left.\begin{matrix} A\ \emptyset \\ \emptyset\ A \end{matrix}\right\} \xrightarrow{w/2} \emptyset\,\emptyset, \qquad A\,\emptyset \overset{D}{\leftrightarrow} \emptyset\,A, \tag{9}$$

where $D = (1-w)/2$. The model at $w=0$ is the tp12. Our numerical results in Fig. 8 show the typical "crossover" behavior between the identical universality class discussed in Sec. II. The critical line converges to the $w=0$ point *linearly* ($\phi = 1$), and there is no diverging time scale as w becomes

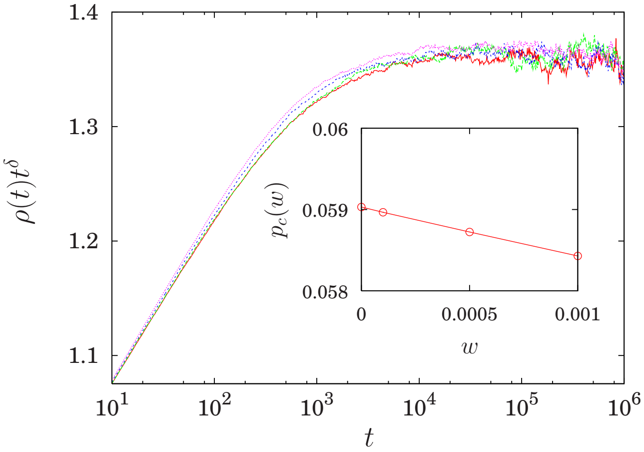

FIG. 8. (Color online) Semilogarithmic plot of $\rho(t)t^{\delta}$ vs t with δ=0.1595 at p_c=0.058 427, 0.058 723, 0.058 966, and 0.059 031 for $w=10^{-3}$, 5×10^{-4}, 10^{-4}, and 0 (from left to right), respectively. There is no diverging time scale as w approaches zero. Inset: phase boundary near w=0. Each symbol corresponds to the critical point used in the main figure. The straight line is drawn between two consecutive points, independently.

smaller with almost perfect collapse of all critical density decay curves near $w\approx 0$. Our crossover study provides another piece of strong numerical evidence that the (diffusive) tp12 belongs to the DP class.

If the PCPD does not belong to the DP class, why should the tp12 belong to the DP class? The PCPD and the tp12 seem quite similar in the sense of their multiparticle nature in reaction dynamics and also the absorbing space structure (vacuum and a single-particle state). However, they are quite different in the role of diffusing isolated particles. See Fig. 9 for the space-time configurations for the tp12 and the PCPD at criticality, starting from the low density initial condition without pairs. The diffusing isolated particles of the tp12 cannot increase the number of particles in most cases, because $2A\rightarrow\emptyset$ dynamics dominates over $3A\rightarrow 4A$ dynamics: Pairs generated by collisions of two isolated particles evaporate before greeting another isolated particle to become "active" triplets. Consequently there is effectively no feedback mechanism from isolated particles to increase the particle density or make the system more active. Therefore the region of isolated particles can be regarded as absorbing like in the PCP model. This case may correspond to the *no-feedback* point (r=0) for the generalized PCPD (GPCPD) [13], which is known as the DP point. This argument can be generalized to systems with hybrid-type reaction dynamics such that $mA\rightarrow(m+k)A$ and $nA\rightarrow(n-l)A$ with $m>n$ and $k,l>0$, which are numerically shown to belong to the DP class [12].

The isolated particles of the PCPD, however, cannot be regarded as absorbing as Fig. 9 shows; the isolated particles

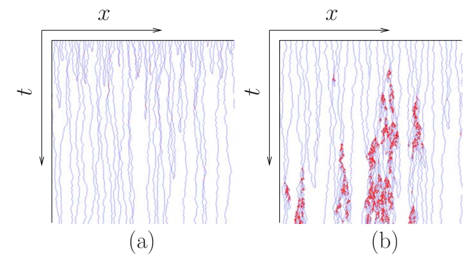

FIG. 9. (Color online) Space-time configurations of (a) the tp12 and (b) the PCPD at criticality. The initial density is (a) $\frac{1}{16}$ for the tp12 and (b) $\frac{1}{32}$ for the PCPD with size $L=2^{10}$. The isolated particles in the PCPD can increase its activity by diffusion whereas those in the tp12 cannot effectively because the $3A\rightarrow 4A$ reaction hardly occurs before $2A\rightarrow\emptyset$ annihilation.

may affect the critical spreading actively, because both dynamics of $2A\rightarrow\emptyset$ and $2A\rightarrow 3A$ compete each other and consequently there is an effective feedback mechanism from diffusing particles to make the system active. This corresponds to the GPCPD with *long-term memory* effects at $r\neq 0$ [13].

VI. SUMMARY AND CONCLUSION

In summary, we studied the crossover from the model belonging to the directed percolation class with infinitely many absorbing states ($\mathrm{DP_{IM}}$) to the DP model by reducing the number of absorbing states significantly. The crossover is found to be well described by the usual crossover scaling function for the order parameter. The crossover exponent ϕ is argued to be related to one of the DP critical exponents—i.e., $\phi=\nu_{\parallel}$—which is further evidenced by the similar crossover model belonging to the directed Ising class. The origin of the diverging scale and the nontrivial crossover comes from the discontinuity of the auxiliary field density at the $\mathrm{DP_{IM}}$ critical point. Our study presents evidence for the existence of nontrivial scaling in the $\mathrm{DP_{IM}}$, which is compared with the study on the spreading exponents.

We also studied how the crossover behavior from the $\mathrm{DP_{IM}}$ to the DP is affected by particle diffusion. The crossover from the pair contact process with diffusion to the DP studied in Ref. [19] is well classified by the nontrivial crossover exponent. On the other hand, the diffusive tp12 which is known to belong to the DP class is characterized by the trivial "crossover" between the identical class detailed in Sec. II. This provides an additional evidence supporting that the PCPD is distinct from the DP. In addition, we suggest an argument based on the role of diffusing isolated particles, explaining why the tp12 should belong to the DP class, but the PCPD does not need to be.

It will be a challenging problem to see if the crossover scaling from the $\mathrm{DP_{IM}}$ to the DP can be anticipated in the framework of the field theory [6].

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